

The Simple Pendulum

According to theory¹, the period of a simple pendulum is independent of mass and is dependent on the square root of its length; i.e. $T = 2\pi\sqrt{L/g}$, where T is the period of the pendulum, L is the length of the pendulum and g is the acceleration of gravity.

A simple pendulum is set up in the laboratory to test this assertion. A vertical shaft is placed in a lab table using the provided receptacle. A horizontal bar is then connected to the vertical shaft with a clamp, and a string is attached to the horizontal bar. A mass is attached to the other end of the string and pulled back and let go, so that it executes (approximately) simple harmonic motion. The time required to complete 30 cycles (t) is measured with a stopwatch and recorded. To improve accuracy, three trials are completed for each measurement. The average of the recorded values of t for the three trials is then divided by 30 to obtain the period (T) of the motion.

In the first part of the experiment, the length of the string is kept constant and different masses are attached to its end to determine the dependence (or lack thereof) of the period on mass. In the second part, the length of the string is varied, while keeping the mass constant in order to determine the dependence of the period on length. Graphs are drawn for period vs. mass and for period vs. length. A graph of $\log T$ vs. $\log L$ is then drawn in order to determine the specific power dependence of T on L . The value obtained for this power is compared to the expected value of $1/2$.

Finally, a graph of T vs. \sqrt{L} is drawn. According to the equation above, the slope of this graph should be $2\pi/\sqrt{g}$. Thus this slope can be used to calculate an experimental value for g , which is then compared to the theoretical value of 9.80 m/s^2 .²

¹Douglas C. Giancoli, Physics, Prentice-Hall, 1998, p. 319

²Ibid., p. 34

Part I: Dependence of Period on Mass

The length (L) of the pendulum is held constant at 30 cm while the mass (m) is varied from 50 g to 500 g, in 50 g increments.

The time required for 30 cycles is measured in three trials and represented by t_1 , t_2 , t_3 . The period (time for one cycle) is represented by T and is calculated from the average value of t: $T = t_{\text{avg}}/30$.

All times are measured in seconds.

The mass is measured in grams, then converted to kilograms.

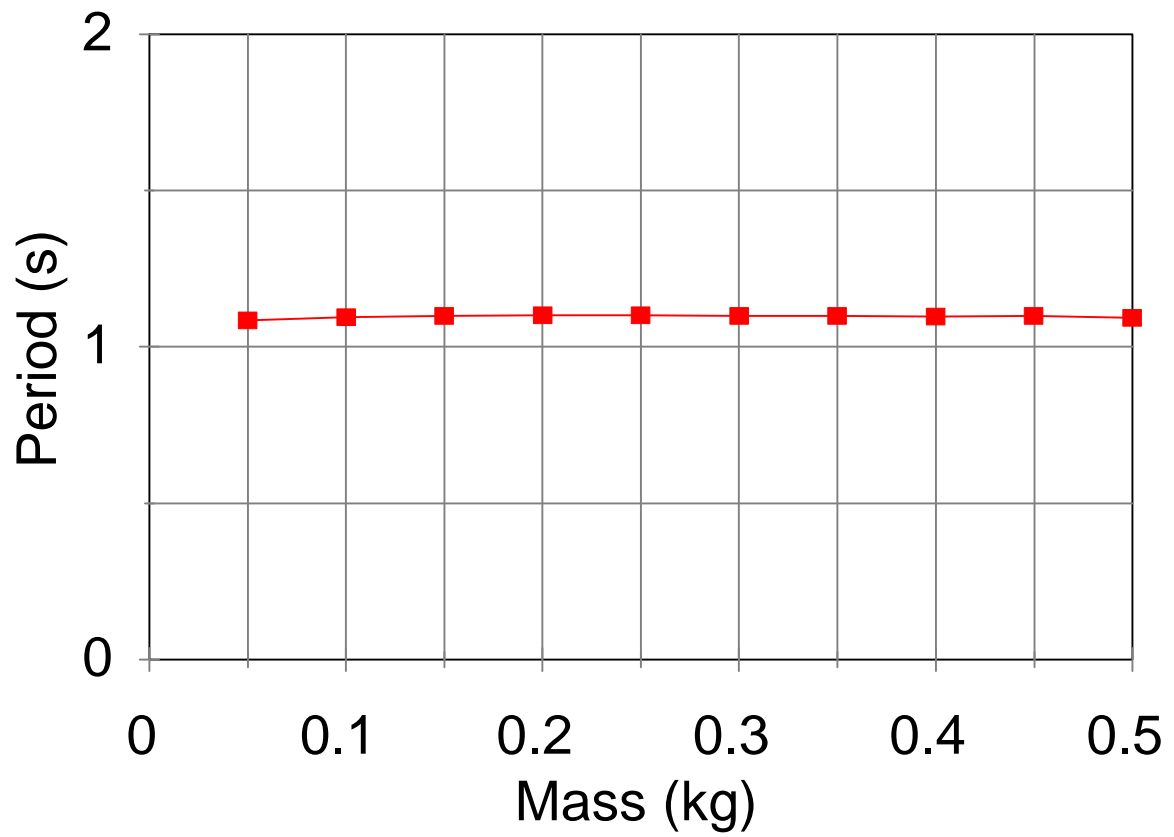
Table 1: Period vs. Mass for Simple Pendulum

t_1 (s)	t_2 (s)	t_3 (s)	t_{avg} (s)	T (s)	m (g)	m (kg)
32.57	32.51	32.53	32.54	1.085	50	.050
32.85	32.90	32.88	32.88	1.096	100	.100
33.01	33.01	33.00	33.01	1.100	150	.150
33.06	33.01	33.06	33.04	1.101	200	.200
32.96	33.11	33.04	33.04	1.101	250	.250
32.98	32.99	33.00	32.99	1.100	300	.300
32.96	32.96	32.96	32.96	1.099	350	.350
32.96	32.96	32.94	32.95	1.098	400	.400
32.94	32.99	32.99	32.97	1.099	450	.450
32.86	32.83	32.83	32.84	1.065	500	.500

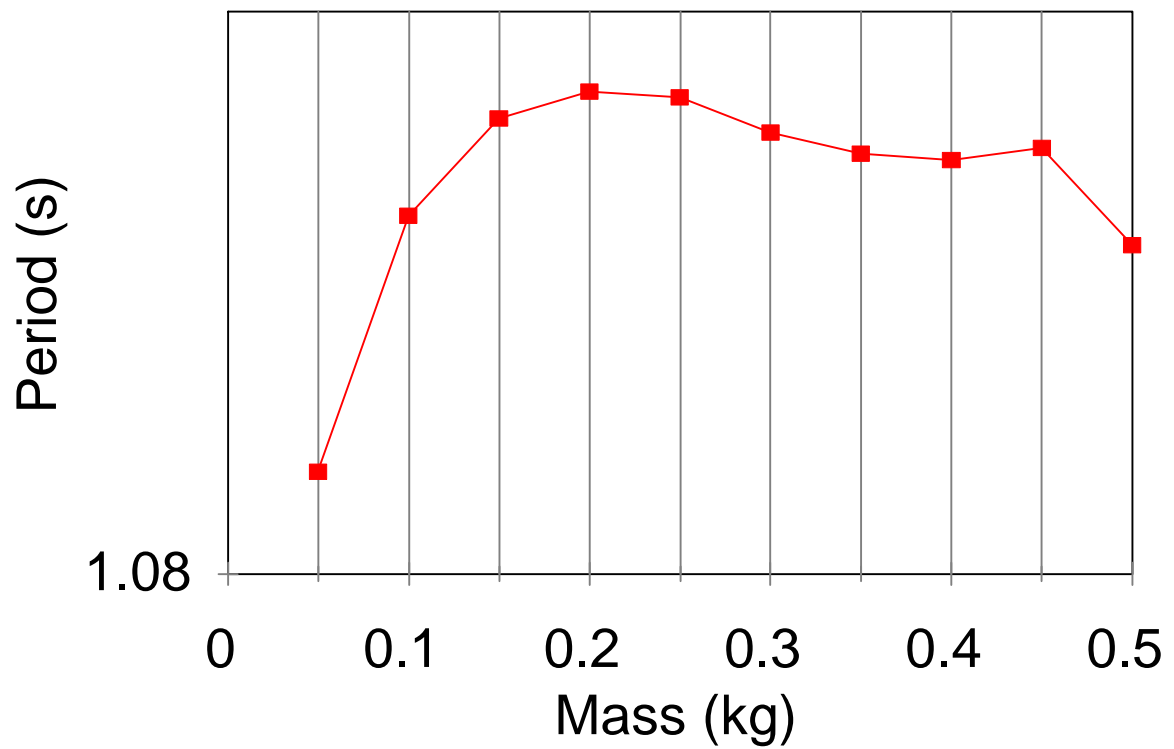
The data for T vs. m from this table is represented on the graph on page 3*. The graph on page 4 shows the same data with an exaggerated scale for the period. We see that the variation in the period is no more than 0.036 s (3.29%).

*All graphs and slope and uncertainty calculations for this report were done using QuatroPro

Period vs. Mass for Simple Pendulum



Period vs. Mass for Simple Pendulum (Exaggerated Scale)



Part II: Dependence of Period on Length

The mass (m) is held constant at 250 g and the length (L) is varied from 10 cm to 50 cm, in 5 cm increments..

Times are measured and period calculated as in part I. As before, all times are in seconds.

Lengths are measured in centimeters, then converted to m.

Table 2: Period vs. Length for Simple Pendulum

t_1 (s)	t_2 (s)	t_3 (s)	t_{avg} (s)	T (s)	L (cm)	L (m)
19.12	19.17	19.17	19.15	.6383	10	.10
23.19	23.30	23.27	23.25	.7751	15	.15
26.97	27.04	27.04	27.02	.9006	20	.20
30.09	30.04	30.16	30.10	1.003	25	.25
33.01	33.06	33.09	33.05	1.102	30	.30
35.73	35.78	35.79	35.77	1.192	35	.35
38.18	38.21	38.16	38.18	1.273	40	.40
40.48	40.34	40.45	40.42	1.347	45	.45
43.06	42.98	42.79	42.95	1.432	50	.50

From the graph on page 6, we see that the relationship between T and L is not linear.

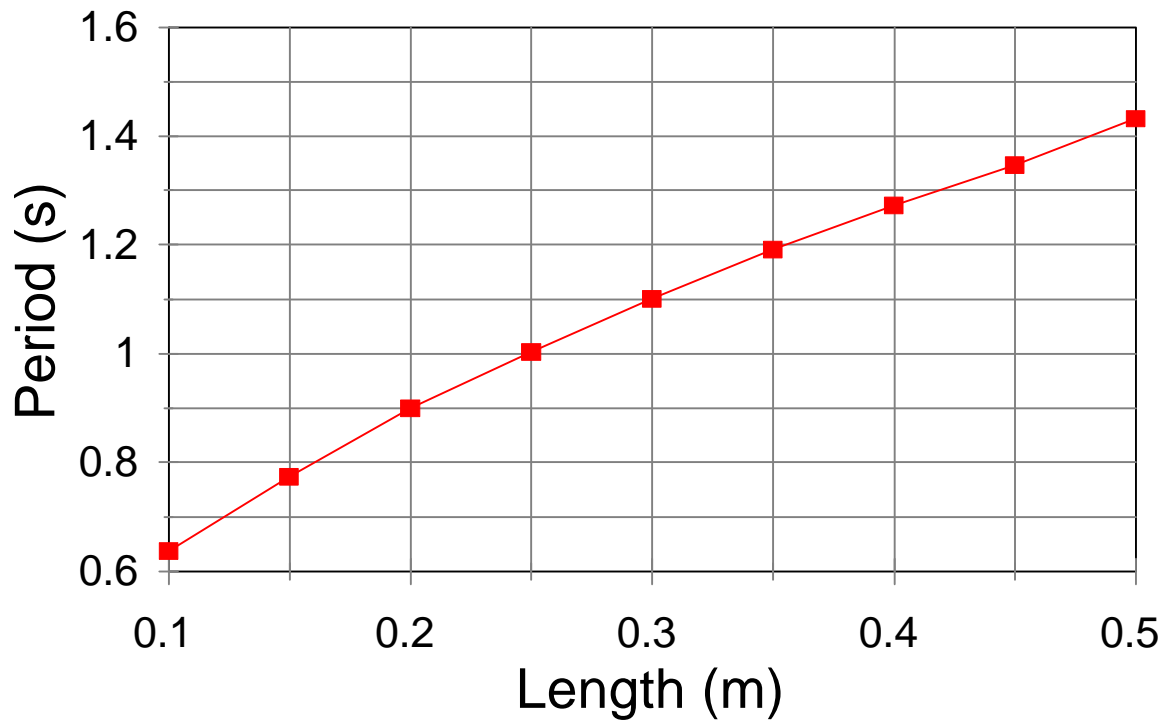
The log T vs. log L graph on page 7 has a slope of $0.509 \pm .0036$, indicating that T is proportional to that power of L.

The error here (as compared to the expected value of 1/2) is

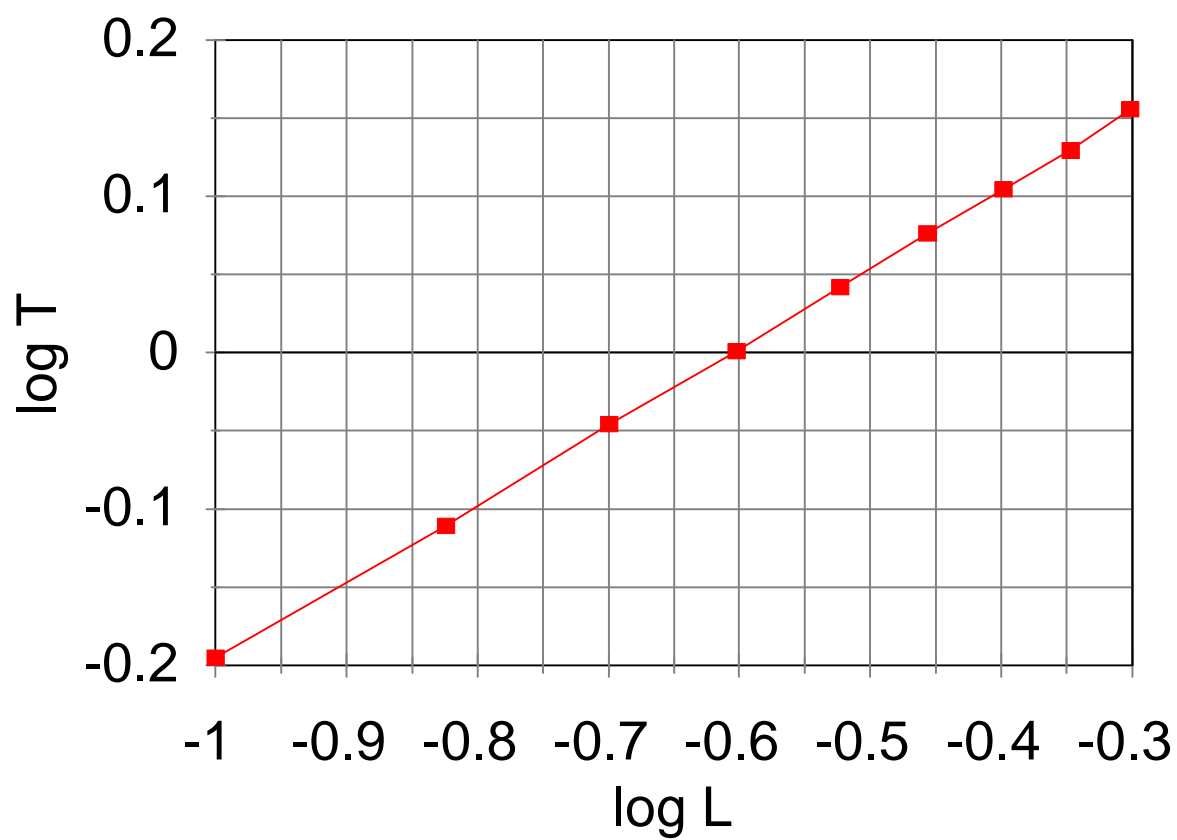
$$[(0.509 - 0.500) / 0.500] \times 100 = 1.8\%$$

The experimental range of values for the slope is $.505 \leq \text{slope} \leq .513$

Period vs. Length for Simple Pendulum



log T vs. log L for Simple Pendulum



Part III: Calculation of g

The graph on page 9 shows the dependence of T on \sqrt{L} .
The slope of this graph is $2.02 \pm 0.0015 \text{ s/m}^{1/2}$.

$$\text{Since slope} = 2\pi/\sqrt{g},$$

$$g = (2\pi/\text{slope})^2$$

$$\text{i.e. } g = (2\pi/2.02)^2 = 9.68 \text{ m/s}^2$$

The error here, compared to the expected value of $g = 9.80 \text{ m/s}^2$ is

$$[(9.80 \text{ m/s}^2 - 9.68 \text{ m/s}^2) / 9.80 \text{ m/s}^2] \times 100 = 1.2\%$$

Taking the uncertainty stated above into account we get a range of g-values given by:

$$9.66 \text{ m/s}^2 \leq g \leq 9.69 \text{ m/s}^2$$

Sources of Error

A small error is introduced by assuming that the motion is simple harmonic. (A real pendulum only approximates simple harmonic motion.) There is some friction in the string where it moves on the metal rod, and there is some air friction. Also, the theoretical value used for g may be slightly off due to local variations in its value. There may be some error introduced by imperfections in the ruler used to measure length and in the stopwatch. An additional error is introduced by the innate variation in human reaction time when operating the stopwatch.

Conclusions

Our data showed the period of our simple pendulum to be independent of mass, within 3.9%, which is consistent with the uncertainty of the measurements made with the stopwatch. (Time is measured to the nearest .01 s, which corresponds to an uncertainty of 0.9% in the period, but we might expect the uncertainty of the measurements to be slightly larger than that since the stopwatch itself is somewhat imperfect and there is some small variation in the reaction time of the experimenter.)

The period was found to be dependent on the 1/2 power of the length, with an error of 1.8%, but the range of values for the power dependence of T on L (taking calculated uncertainty into account) does not quite include the expected value of 1/2. Our value of g was within 1.2% of the theoretical value, but again the range of experimental values obtained for g does not quite include the expected experimental value. These differences are small and can probably be explained by some of the small errors listed above, which were not necessarily accounted for in the uncertainty calculations. Hence our results seem to be in agreement with existing theory, but in order to determine this for certain, we would need to do further investigation, attempting to control some of these variables.

Period vs. Square Root of Length for Simple Pendulum

